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Published in the United States of America

To Genevee, my first finance instructor and the first entrepreneur that I ever knew, who is also my mother





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Finance textbooks in general possess a uniqueness all their own, especially given recent developments on the global economic landscape. Textbooks on personal finance, though, have lagged behind considerably from their traditional finance counterparts. The meltdown of 2008 shined the light on the greed of crooked executives on Wall Street for sure; however, very little has been discussed concerning the general public's role in the fiasco: a basic lack of financial literacy. Granted, a detailed understanding of securities and derivatives is not expected of the average citizen or college student, but basic knowledge concerning home ownership, mortgage types, and budgeting is. Sensing this lack, I sought out to do what I could within the confines of my own world of mathematics. On my campus, I developed two courses, one titled **The Mathematics of Personal Finance** and the other titled **The Mathematics of Financial Life Management**. Whereas the first course is a mathematically based honors course in personal finance, the second one is a similar version available to all students.

On one hand, what motivated the textbook is simple. While developing the courses, I struggled to find a book whose content matched exactly what I wanted to cover in the classroom. After perusing for months, I finally settled on one and had students purchase it. When the semester began, students faithfully brought the text to class, but I rarely ever used it. I generally posted my own notes online to which students referred. At almost every class meeting, I began by apologizing for not using the text for the current class session and began flipping through my trusted set of notes. After about my fourth apology, a student finally inquired, "Professor, why don't you just write your own book for the course? You already have all the notes for it." I must have stopped writing on the board mid-equation because her suggestion hit me like a ton of bricks. I stared at her and responded, "You know, you're absolutely right! Why don't I?" So I did. Thanks, Annie Jaffrey!

On a deeper level, this text emanates from the same source of origination as the course itself: my sincere concern for the level of basic financial illiteracy among college students. After watching Ferguson's *Inside Job*, I was discouraged by how the common person bore the brunt of fund mishandling. After I saw the documentary *Maxed Out*, I realized how shackled many people feel with respect to consumer

#### Preface

credit and its effect on one's lifestyle. Finally, as I pass through campus on a daily basis, I notice very few mechanisms in place to educate the next generation about money management and its role in fostering a healthy spending and saving diet. This issue surely is not confined to the walls of my institution. Banks continue to seek college students at an alarming rate in an effort to make good on new prospects in the dash to award fresh credit. Students across the country and beyond have to use loans to finance their education and then learn how to balance their own personal finances once they graduate. It becomes an egregious disservice to them if academe drops the ball by not empowering them in some way to strengthen their financial selves in addition to preparing them in their more traditional academic majors. This textbook aims at fulfilling that vision of empowering students through basic financial knowledge.

To play on the whole financial theme throughout the text, "Chapters" are referred to as "Assets," which is a key term in finance that means anything owned having positive economic value. By no means is this another one of those *Personal Finance for Dummies* texts. It assumes a basic understanding of algebra. In fact, Asset One provides a quick review of algebraic fundamentals that are key to grasping basic ideas in personal finance. In Asset Two, the notion of interest is discussed and the many different ways it rears its head in the world of personal finance through several basic savings strategies.

Asset Three discusses the mathematics behind credit cards and owning a home. We address credit scores as well as paying back student loans. This is a particularly interesting topic for college students, as many of them have at least one credit card that they are learning to incorporate into their financial lives. We discuss benefits and challenges related to owning a home versus renting an apartment and the benefits and challenges of both.

Asset Four addresses the ideas behind investing for retirement. We distinguish between savings in this regard as opposed to the type of savings in Asset Two. We also address how significant it becomes for college students to start saving for retirement in their twenties as opposed to waiting until later.

Taxation and budgeting are key topics in Asset Five. How much of a budget should go toward housing? transportation? food? We challenge college students to learn how to perfect budgeting before landing that dream job. We also help students understand tax brackets and offer suggestions for putting themselves in the best position when tax time comes.

Assets Six and Seven shift to discussing personal finance within the context of statistics. Fortunately, this topic may be navigated without a deep understanding of algebra, but it has its own set of challenges and subtleties. Asset Six helps convey finance through the eyes of simple and multiple linear regressions, with special emphasis on the idea of correlation. Asset Seven discusses the notion of stocks and their relation to time series.

I intersperse some lessons in Excel to help students understand the benefits of using software to reinforce the theory they have learned. Being familiar with Excel poises college students to be a cut above their competition as they pursue careers and become entrepreneurs in their own right.

Finally, for those who wish to appreciate this textbook without subjecting themselves to the mathematics presented, we have placed articles on select financial topics at the end of each asset; each article is entitled *MoneyTalk*. One may read these articles without feeling any mathematical anxiety. There are no equations, no mathematics, and no stress. These articles simply provide a break from the academic slant of the text and allow for a bit of casual financial reading. Such topics explored in these articles include combatting identity theft, the plight of social security, and why having a will is a must.

My hope is that this textbook would serve as a spark in the lives of college students and pique their interest to pursue an understanding of the presented concepts even more. Through a deliberate and joint effort, students can be equipped with the weapons they need to combine their love for money with their passion for mathematics or what I call *Moneymatics*.



No material in this textbook is interpreted as the author's providing financial, investment, legal, accounting, or any related type of advice. The author does not hold degrees or certification in finance. Information included in the textbook are for general purposes and may not be appropriate for your particular financial situation. There is no expectation that results discussed herein, good or bad, will be duplicated in your financial state of affairs. If expert advice is required, you should consult with a financial expert or certified public accountant. Although the author and publisher have done their best to prepare this textbook, they disclaim any liability concerning accuracy or entirety of included information.





Several curricular innovations focus on financial literacy and personal finance exist for grades K-12. The Jump\$tart Coalition for Personal Financial Literacy are among the leaders on the educational scene who promote the National Standards for Financial Literacy. Some curriculum on financial literacy exists for college students; however, the number of financial-literacy courses in departments on college campuses across America remains depressingly low.

Following is a guide for developing such a course in financial literacy on the college level. Mathematics departments or economics departments may offer it. Consistent with an understanding of this textbook, the course assumes sufficient proficiency in algebra. Modules are based on material presented in this textbook, but instructors may tweak according to their liking. The course assumes a 15-week semester, in which four hours per week are dedicated for the course (ideally two hours per meeting, two days a week). Two exams and a cumulative final exam are suggested. A more advanced course may incorporate course projects, in which students research a topic in finance and present findings via PowerPoint or Prezi to the class. For a course project also, students may read separate books on a topic in finance and write a response paper, analyzing the book's strengths and shortcomings.

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Developing a Mathematically Based Course in Financial Literacy at the College Level

# **A Course Proposal**

Suggested	Course	Title
Prerequisit	te	
Suggested	Course	Credits

The Mathematics of Personal Finance Algebra proficiency 4 hours per week; 3 credits (2 hours lecture, 2 hours lab)

# **Course Rationale**

Basic literacy in personal finance remains a constant challenge for today's college student. Without adequate knowledge in this respect, students run the risk of damaging their financial well-being and destroying their financial health. This could haunt them long after graduating from college. Recent assessment conducted by the Organization for Economic Cooperation and Development revealed that students in the United States scored below the average of the 18 countries evaluated; results also exposed below-average rankings for the United States as they relate to the percentage of students who have a baseline level of competence regarding personal finances. All relevant disciplines, including mathematics, have a role and responsibility to address this dire reality in their own unique style.

# **Course Description**

Primarily, this course is designed to strengthen students' mathematical core using algebraically based techniques associated with topics in personal finance. Further, students use computer software to make practical sense of financially based mathematical formulas and to analyze related results visually through graphs and charts. Finally, the aim of the course is to increase students' baseline competence in select areas of personal finance. Although some of the topics in the course have been on the financial scene for a long time (e.g., interest, annuities, budgeting), other areas of exploration serve as fairly new developments on the financial landscape (e.g., PEG ratios, forecasting methods for time series). Using advanced topics in mathematics and statistics, students gain primary insight into select topics in personal finance by performing comparative analyses and by scrutinizing software results. Several of these mathematical topics include annuity analysis, regression analysis, and time series. Through discussing articles, watching relevant videos, and listening to guest speakers, students are additionally exposed to particulars that fortify their understanding of salient topics in personal finance.

# **Catalog Description**

Uses advanced mathematical and statistical techniques to analyze select topics in personal finance. Includes exploration into annuity analysis, regression methods, and time series analysis. Application areas involve managing credit cards, paying back student loans, and choosing a stock. Uses computers to analyze and interpret financially based data.

# **Course Objectives**

Upon completion of this course, students will be able to:

- Distinguish between interest types used for various kinds of savings accounts
- Compare the effects of various interest rates on retirement accounts, mortgages, and credit cards

- Complete a budget form based on personal financial situations
- Discuss in writing pros and cons associated with renting versus buying property
- Show savings obtained from different credit scores for the same purchase
- Determine how much in federal and state taxes are owed in simple tax situations
- Demonstrate techniques associated with simple linear regression techniques using personal finance data
- Define what a stock is and master how to choose a stock based on PEG ratios
- Predict stock prices and smooth series using the latest in time series methods

# **Course Textbook**

Dillard, Bernard L. 2017. Moneymatics: Where Money and Mathematics Meet. Dubuque, IA: Kendall Hunt.

# Suggested Course Assessment

The final course grade may be determined as follows:

- 1. Labs and In-Class Assignments (20%). Excel exercises and basic problem sets are to be turned in during class. No makeup chance is suggested. This serves as basic class participation.
- 2. Quizzes (15%). Four quizzes are suggested with the lowest quiz grade dropped. Quizzes should be no longer than 25 minutes.
- **3.** Exam 1 (20%). This two-hour exam is based on Module 1 through Module 9 (partial). The exam should evaluate mathematical proficiency as well as material discussed in articles and in videos.
- **4. Exam 2** (20%). This two-hour exam is based on Module 9 (partial) through Module 15. The exam should evaluate mathematical proficiency as well as material discussed in articles and in videos.
- 5. Final Exam (25%). This is an exam that *must* cover material from all Modules.

# Learning Units of Study

(Note: Instructors are strongly advised to contact financial-literacy foundations to have volunteer guest speakers take the lead on Unit 10. A basic online search should reveal volunteer agencies in your area that provide basic financial-literacy education to colleges and universities.)

# Unit 1 Money: Then and Now

# 1 hour

This serves as the opening unit for the course. The unit introduces the notion of money and provides a working definition for money. The unit focuses on a brief history of money and how bartering existed before banknotes became the exchange standard. Emphasis is placed on the progression of the currency form from bartering to representative money to metal coins to fiat money. The unit addresses when the first paper money was issued in the United States. Key terms to discuss are as follows: currency, money, barter, and representative money.

#### xiv Developing a Mathematically Based Course in Financial Literacy at the College Level

- Recommended Video on the History of Money •
  - Outlearn on YouTube. "The History of Money (combined)." YouTube video. YouTube, December 1, 2015. Accessed August 15, 2016. https://goo.gl/Zr4dDl.

#### Unit 2 **Interest and Savings**

This unit focuses on the basic notion of interest. Instructors should have students read in class an article on interest and three key savings programs. Students are expected to highlight notable differences between the regular savings account, the certificate of deposit (CD), and the money market account (MMA). Instructors should introduce what Federal Deposit Insurance Corporation (FDIC) means and discuss its significance in relation to savings accounts. Instructors should supplement the discussion with a video that summarizes key points. Several articles and videos exist; however, some acceptable suggestions follow:

- Recommended Article on Interest and the Three Savings Instruments Taylor, Joe. "Compare Savings Accounts, Money Market Accounts, and CDs." Money Rates.com. Accessed August 7, 2016. http://goo.gl/7PmJZJ.
- Recommended Video on Savings Accounts, CDs, and MMAs Personal Finance in the Elementary Grades. "4.2 Savings Accounts." YouTube video. YouTube, August 15, 2014. Accessed August 7, 2016. https://goo.gl/B3uHlW.

#### **Linear Equations and Exponential Functions** Unit 3

This unit introduces linear equations. Students demonstrate how to find the equation of a line, given two points. Only examples within the context of personal finance should be used (e.g., hours worked as x and income received as y). Students master the concept of slope and y intercept and demonstrate how to graph the line using only those two values. Students work primarily with the slope-intercept form of the line.

The unit transitions to a discussion of exponential equations. Examples to motivate exponential functions is the historic one in which the student saves a penny and then doubles it each day and continues the pattern for 30 days (two cents, four cents, eight cents, etc.), giving note to how much money has been saved by the end of the month. Students write conceptual differences between linear equation and exponential functions. Students master finding the base and the coefficient for exponential functions. Students graph exponential functions and compare to graph of linear equations over time, noting visual differences.

#### Simple and Compound Interest Unit 4

This unit explores the mathematics behind simple and compound interest. The unit stresses the major difference between simple and compound interest. Students show parallels between the formula for simple interest and linear equations; students also show parallels between the formula for compound interest and exponential equations. Students master how to solve iteratively for each variable in both interest formulas. Students learn how to apply properties of logarithms. Students will be able to define the following key terms: interest, principal, and interest rate.

# 4 hours

2 hours

2 hours

Students work in Excel to show how money grows over time in simple and compound interest with the same initial deposit, interest rate, and time. When programming, students discover how to exploit absolute and relative cell range references. Students graph these results in Excel, observing which interest type generally outperforms the other over time. Students use Excel to write formulas that solve for each variable in terms of the others for the two interest types.

# Unit 5 Credit and the Credit Score

This unit is dedicated to discussing credit and its role in personal finance. Instructors introduce the term *credit* and the idea of the credit score. To motivate the notion, the credit score and one's personal finance history are discussed in relation to a college student's grade point average (GPA) and their academic past. Students research what Fair Isaac Corporation (FICO) stands for. They must be able to state and discuss in writing the five elements of a FICO credit score and what percentage each element contributes to the score. Instructors define what a credit card is and tie to the discussion of interest from the previous unit. Students may read about the five elements. Students should be encouraged to visit sites like www.creditkarma.com to locate their credit score for free. Students should be advised that they will have to enter their social security number but should not enter any credit card information to secure score. Implications of the credit score will be addressed in a future unit. Key terms to discuss are the following: minimum payment, credit card perk, annual fee, due date, and penalty. A recommended video about credit scores follows.

Recommended Video on the Credit Score and Its 5 Elements

Federal Reserve Bank of St. Louis. "Understanding How a FICO Credit Score is Determined." YouTube video. YouTube, January 24, 2015. Accessed May 6, 2017. https://goo.gl/tUOjN4.

# Unit 6 Present Value of an Annuity: Credit Cards

This unit introduces mathematical content that provides the backdrop for an understanding of credit cards. Instructors define annuity and introduce the formula for the present value of an annuity. Instructors conceptually distinguish between this formula and savings formulas introduced in Unit 4. Using the calculator, students practice solving iteratively for each variable in the equation, especially solving for time using logarithmic properties. Key terms to discuss are the following: annuity, present value of an annuity, and compounding period.

This unit further explores scenarios involving making the minimum monthly payment for credit card bills versus paying more than the minimum per payment. Students use the present value formula to compare how much time is saved in paying off a credit card balance when the same minimum payment is made each month versus when the same more-than-minimum amount is paid per month. Instructors discuss and show examples related to possible traps associated with credit cards. Instructors also discuss how these concepts may be used to the consumer's advantage through discipline. The following key terms are discussed: credit card perk, balance transfer, teaser rate, and deferred interest.

The Excel exercise in this unit should give students practice showing the paying down of a credit card each month making minimum payments. Instructors should also make students use Excel to

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# 5 hours

# 2 hours

show how doubling the payment affects time it takes to pay the credit card balance off. Students should also be able to determine in Excel whether it saves them more money to transfer a balance from one card to the next.

• Recommended Video on Credit Cards

Keen, Trevor. "FUNNY MONEY – Get it on Credit."YouTube video. YouTube, May 16, 2013. Accessed May 6, 2017. https://goo.gl/IEXZNl.

# Unit 7 Present Value of an Annuity: Student Loans

This unit continues discussion of the present value of an annuity as applied to student loans. This unit is more personalized with students since most will probably have current student loans. Students verbalize the major difference between subsidized/unsubsidized loans and consolidated loans (i.e., interest rate based on credit score or not). If students have student loans, they must research the interest rate. Students calculate interest on subsidized student loans, assuming the six-month grace period has ended after their graduation. Instructors introduce the notion of loan consolidation, discussing pros and cons associated with it. Students determine mathematically whether it makes sense to consolidate several loans into one, given a set time to pay it off, certain interest rates, and origination fees tied to consolidation. That is, students calculate the total interest saved for payoff of a consolidated loan and compare to total interest saved for payoff of separate student and private loans. The unit presents problems for which loan consolidation mathematically makes sense and when it does not. Special emphasis explores situations when the time-to-loan payoff for current separate loans is shorter than the time to payoff for the consolidated loan even though the latter may promise a lower interest rate. Key terms to explore are the following: subsidized loan, unsubsidized loan, loan consolidation, origination fee, and loan default.

 MANDATORY Video to Watch on Student Loans and Credit iGrad. "Financial Literacy in College | Financial Literacy Video."YouTube video. YouTube, December 16, 2011. Accessed August 14, 2016. https://goo.gl/G93tjN.

## Unit 8 Renting versus Buying

# 2 hours

3 hours

This unit explores renting and owning property. The purpose of this model is for students to use class time to research and report to fellow students the pros and cons associated with both ideas. This unit stresses no mathematical techniques proper but is dedicated to strengthening students' ability to work together, gather data, and present results. The instructor does *not* teach this topic in the typical lecture format. Students are expected to take the lead. Students learn how to work in groups within time constraints to locate, synthesize, analyze, and present information without instructors' assistance. In groups, students may use any creative means possible to present findings (e.g., presentation at the board, skits). The unit concludes with a showing of a comprehensive video clip to summarize pros and cons, which may address material that students overlooked. One possible video suggestion follows.

• Recommended Video on Renting versus Buying

Bank of America. "Is Buying a Home Right for You?" Online video clip. *Better Money Habits*. Bank of America Corporation in Partnership with Khan Academy. Accessed August 10, 2016. https://goo.gl/JtJ1rd.

#### Unit 9 Present Value of an Annuity: Mortgages

This unit works through the present value of annuity formula applied to mortgages. One aim of this unit is to use mathematics to show comparatively how much money students save on the purchase of the same home but for different interest rates. The notion of the credit score is reintroduced from a perspective of securing a lower interest rate for a higher score. Students calculate the monthly payments for the higher interest rate (lower credit score) and the lower interest rate (higher credit score). Students show the amount of interest paid back in both scenarios and the total amount of interest saved overall, thus revealing the value of having a high credit score. Another aim of this unit is to use mathematics to show comparatively the value of financing a mortgage for a shorter period of time. Students use the formula to show how much is saved in interest when a mortgage is financed for one time period compared to a shorter period of time. Finding the monthly payments associated with both, students calculate the amount of interest paid in each scenario.

This unit then discusses amortization schedules. Students calculate each of the four columns, showing the paying-down effect of loan repayment. Students also learn how to calculate columns when a down payment is involved. Key terms to address in this unit are: fixed, variable, down payment, and amortization schedule.

In Excel, students practice creating a spreadsheet table solving for monthly mortgages payments when the interest rate and time varies. Students should also create amortization schedules in Excel, showing the remaining mortgage balance dwindling to zero.

#### **Unit 10** Budgeting

This unit addresses budgeting and managing money. Coverage includes discussion of percentage recommendations for budget components (e.g., 30% for housing), according to the College Board. Students practice completing a budget and applying the recommended percentages, assuming an initial dollar amount. Hence, this unit helps students understand percentages clearer. Students understand budgets from the perspective of the time element and the budget's goal. Instructors develop a list on the board that students construct, which highlights daily steps students may take to decrease spending in a budget (e.g., walk instead of catching a cab for five blocks). Students are exposed to paper-and-pencil budget sheets as well as free, modern-day budget programs found online and on mobile apps. Students are encouraged to use budget sheets on a regular basis. Instructors may assign reading in addition to coverage in textbook and may incorporate the watching of a video clip on budgeting. Key terms in budget discussion include the following: need, want, budget, percent, projected budget, and actual budget.

- Recommended Article on Budgeting "Budgeting 101." BestColleges.com. Accessed August 10, 2016. http://goo.gl/xbM2qE.
- Recommended Video on Budgeting

Mountain America Credit Union. "Budgeting for College and School Activities." YouTube video. YouTube, October 7, 2015. Accessed August 10, 2016. https://goo.gl/7xdk0p.

#### Unit 11 My Paycheck and Basic Taxes

(Instructors should note that **no** information in this unit should be used as tax advice to students. Students should seek a certified public accountant for professional tax advisement.)

#### 2 hours

2 hours

# 6 hours

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Students observe a sample pay stub and notice key deductions on it. Students calculate percentages on a paycheck that are deducted for Medicare benefits and also for social security. Students verify these percentage deductions on a reputable website. Students use mathematics to calculate how much in federal and state taxes is owed when the standard deduction is used. With the calculator, students use current federal schedules and agreed-upon state schedules to accomplish the task.

The unit further discusses taxes owed after basic pretax deductions and basic itemized deductions. Using an online tax calculator (such as http://goo.gl/cfqb2m or any other reputable one), students calculate how much is owed in taxes, given certain basic financial data provided by the instructor. Students observe how much is saved in taxes owed when pretax deductions and/or itemized deductions are considered. Students become familiar with the following terms: taxes, gross pay, net pay, Federal Insurance Contributions Act (FICA), deduction, pretax deduction, and itemized deduction.

#### Unit 12 Future Value of an Annuity: Retirement

## 6 hours

This unit introduces retirement and the future value of an annuity. Instructors begin with a discussion of retirement in general. Instructors revisit the notion of interest and discuss how it can be used positively for consumers as opposed to negatively in the case of credit cards and student loans. Students are introduced to the following key terms: retirement, individual retirement account (IRA), traditional IRA, Roth IRA, and the time value of money.

Students discover the conceptual difference between the future value of an annuity and the present value of an annuity with respect to a "lump sum." The unit introduces the formula used in futurevalue problems and stresses how to solve for each variable in terms of the other variables. Specific emphasis is placed on algebraic properties, including use of the logarithm. The unit uses mathematics to comparatively show how much is in a retirement account when only the time variable changes and even when the amount invested is considerably less for the longer investment period. Instructors highlight how much more money is in the account when one starts contributing to the account at different ages. Students calculate how much interest accrues to the account in all scenarios. Finally, students use mathematics to calculate the age at which they would need to start investing to reach their financial retirement goals, solving for the time variable in the formula.

The Excel exercise simulates one's retirement account as it is drawn down during retirement. Starting with a lump sum of students' choosing and the amount of yearly income required, the account should be drawn down for 35 years with students making adjustments based on the target retirement period.

# Unit 13 Stocks and Shares

#### 4 hours

This unit discusses stocks, shares, and basic related concepts. Students distinguish between a stock and a share. Students understand the purpose behind stocks and how investment savings can increase (or decrease) at a quicker rate with stocks when compared to other financial savings instruments. Risk tolerance is introduced, along with the concepts of the retirement portfolio and diversification (eggs-in-more-than-one-basket scenario). With respect to diversification, instructors show charts or graphs of historical examples of a portfolio's behavior when only stocks are invested (one basket) versus stocks and mutual funds (two baskets) versus stocks and mutual funds and bonds (three baskets). Students discover how the portfolio's value increases more (generally) in the latter case. To illustrate the idea of stocks' ability to lose money briefly but gain more money over time, instructors may show

a zoomed-in decreasing trend of a certain time period of a retirement portfolio but overall increasing trend of the same portfolio when zooming out. The unit introduces Vanguard's suggested asset percentage allocations by age. Using them, students use mathematics to calculate how much of their retirement funds should be invested into separate allocation categories. Key terms to define are as follows: stock, share, ticker symbol (stock symbol), risk tolerance, retirement portfolio, and diversification.

The unit transitions into discussing one method for choosing a stock to invest in. The instructor strategically chooses four stocks to illustrate this method. Instructors motivate the idea by discussing growth capacity. Students are introduced to the PEG ratio as the stock-choosing strategy. The instructor goes through each phase of arriving at the PEG ratio, highlighting which of the four stocks are the "best" to choose at different phases of the process. That is, students observe which stock is the best when only considering price per share (PPS). Students calculate the price-to-earnings (P/E) ratio of chosen stocks, observing which stock now is the "best," discussing earnings per share (EPS) at this phase. Finally, students calculate the PEG ratio and observe which of the stocks is the "best" at this phase. An ideal choice of a stock when teaching this topic is one that has the highest PPS but has a PEG ratio closer to zero. Using this mathematical process, students discover that just because a stock is expensive does not mean it is not the best one to buy. Students discuss in writing the meaning behind different PEG ratio values and their implications for a stock's predicted growth. Key terms are as follows: growth capacity, price per share, earnings per share, P/E ratio, and PEG ratio.

# Unit 14 Stocks and Time Series

This unit explores stocks in relation to time series. The concept of time series is defined, along with the four components that comprise a time series. Students demonstrate how to smooth a time series using the simple moving average (SMA) and the exponential weighted moving average (EWMA). They verbalize differences between both conceptually. Students also demonstrate how to predict stock prices one time period outside of the given data range of the stock's prices using EWMA techniques. Students further use Excel regression techniques to predict a stock's value one time period outside of the stock price's data range. Using the *Data Analysis ToolPak*, students show how to adjust the explanatory-variable column to make regression techniques meaningful when using time series. Students write interpretations for the slope and *y*-intercept within the context of time series. Students model how to plot time series of original data, SMA values, and EWMA values on the same plot.

Students use Excel to practice smoothing and plotting time series. Students are given an appropriate set of financial data (e.g., closing stock values for the Dow Jones Industrial Average) and comment on trend and seasonal components. They create columns for an SMA and an EWMA and plot original series and the other two on the same plot. They also perform a regression on the original data after adjusting explanatory column and write the equation for the regression line, interpreting meanings of the slope and *y*-intercept. They predict one time period outside of the given range.

# Unit 15 Simple Linear Regression

#### 5 hours

5 hours

This unit introduces simple linear regression. Instructors review basic concepts of a line and lead into discussion of the least-squares regression line, highlighting the meaning behind bivariate data. Explanatory (independent) and response (dependent) variables are examined. When applied to a

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problem, examples of both variables are only discussed within the context of personal finance (e.g., number of credit cards owned as explanatory and credit score as response). Students learn how to construct scatterplots and understand how to find and draw the regression line best fitting the data. In the process, students define slope and *y*-intercept and solve for both by hand using appropriate formulas. Students must write the interpretation for the meanings of the slope and *y*-intercept within the context of the financial world of the problem. Students predict the response variable given a value for the explanatory variable. Students also describe why the least-squares regression line is called such. Further, students define correlation coefficient and coefficient of determination; they write the interpretation for the meanings of both within the financial context.

Excel tests students' ability to construct formulas to find the slope and *y*-intercept in simple linear regression. Students use the formulas for both as starting points and build numerators and denominators through their programming skills. Students also create a scatterplot from the data and superimpose the regression line on the same graph, placing the equation as well as the coefficient of determination on it. Using the *Data Analysis ToolPak*, students show how Excel renders results. Students locate slope, *y*-intercept, correlation coefficient, and coefficient of determination on Excel results.

Learning Units of Study and Suggested Hours					
Week	Unit	Hours			
1	Course Introduction; Units 1 and 2	4			
2	Units 3 and 4	4			
3	Units 4 (cont.) and Unit 5	4			
4	Unit 6	4			
5	Units 6 (cont.) and 7	4			
6	Unit 8 and <i>Exam 1</i>	4			
7	Unit 9	4			
8	Units 9 (cont.) and 10	4			
9	Units 11 and 12	4			
10	Unit 12 (cont.)	4			
11	Unit 13	4			
12	Unit 14	4			
13	Units 14 (cont.) and 15 and <i>Exam 2</i>	4			
14	Unit 15 (cont.)	4			
15	Review for Final Exam and <i>Final Exam</i>	4			
	Total hours	60			



BERNARD L. DILLARD is an associate professor of mathematics at Fashion Institute of Technology in New York City. He enjoys teaching a variety of courses, including Statistical Analysis, Data Analysis for Business Applications, and The Mathematics of Personal Finance. He is the coauthor of *Elementary Statistics (4<sup>th</sup> edition)*, also published by Kendall Hunt. Away from mathematics, Bernard enjoys acting, modeling, traveling, and watching independent films.

