# LESSON 2

# Interest Formulas and Their Applications



## Overview of Interest Formulas and Their Applications

Several important principles in engineering economy, as well as concepts and key factors in problem-solving processes, are introduced in Lesson 2. Those key terms are extremely important to understand since they will be predominantly utilized for the more complicated engineering economy problems, especially in the economic analysis of alternatives presented in Lessons 3 and 4. Each symbol used in the formulas and equations should be memorized beforehand. The first interest formula is the starting point for the more aggressive problem-solving processes. Additionally, it is the basic factor for the derivation of several more advanced formulas. Details of the derivation of each of the interest formulas will not be presented in this book, except for the very first formula. As the main purpose of this book, its practical applications will be presented and thoroughly explored. The concept of the time value of money will be extensively applied from this lesson on. The most common mistake in solving engineering economy problems has been found in the compounding interest calculation due to the use of inappropriate factors. However, Table 2.1 presents comprehensive examples to avoid such mistakes. The difference between the annual percentage rate (APR) and the annual percentage yield (APY) is explained here in terms of the nominal and effective rates of interest. Finding an unknown value in any equivalent cash flow diagram can be challenging and complicated. Those challenging and/or complicated problem-solving tasks will become easy and simple through an introduction of a unique procedure introduced at the end of this lesson along with several typical examples. Finally, three sets of comprehensive review exercises are provided at the end of this lesson. It is extremely important that you attentively explore them all by yourself to understand the principles and key concepts learned from this extended lesson.

## Symbols Used in Engineering Economy

The following symbols will be used throughout the book:

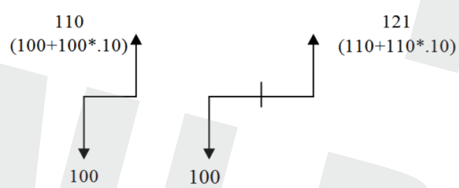
- ***P*** = present amount
- ***i*** = interest rate per interest period
- ***n*** = number of periods
- ***F*** = future amount, equal to the *P* present amount after *n* time periods given interest rate *i*
- ***A*** = a uniform end-of-period cash inflow or outflow, continuing for *n* periods
- ***G*** = a cash flow that increases or decreases by a uniform amount each period (the arithmetic gradient)

- ***r*** = nominal interest rate per year (APR)
- ***m*** = number of compounding subperiods per year
- ***i**** = effective rate of interest per year (APY)

It is important to remember that *P* does not always mean present, and *F* does not always mean future. These symbols are relative to each other on the timeline. *P* always occurs earlier in time than *F*.

## Derivation of the Interest Formula

Suppose you invest $100. How much will you have if $i = 10\%$ after one year? Two years?




Generalizing with P=P, i=i, $F_1, F_2, F_3$

$$F_1 = P + Pi = P(1+i)$$
$$F_2 = F_1 + F_1 i = P(1+i) + P(1+i)i$$
$$= P(1+i)(1+i) = P(1+i)^2$$
$$F_3 = F_2 + F_2 i = P(1+i)^2 + P(1+i)^2 i$$
$$= P(1+i)^2(1+i) = P(1+i)^3$$

$$F_n = P(1+i)^n$$

## Interest Tables

### Basic Concept

Algorithms have previously been calculated for solving interest rate problems. Take advantage of these by using the interest tables in the back of the book.

### Details

To find the future value of money, *F*, given the initial value, *P*, from *n* years ago:

- To find *F* given *P*, refer to the interest table for the given rate and locate the proper number of years under the *F/P* heading. Multiply the *P* by this number (the part in the parentheses is from the interest table) to find *F* as is shown in the following formula:

$$F = P(F/P, i\%, n)$$

To find the initial value, *P*, given the final value, *F*:

- To find *P* given *F*, refer to the interest table for the given rate and locate the proper number of years under the *P/F* heading. Multiply *F* by this number (the part in parentheses is from the interest table) to find *P* as is shown in the following formula:

$$P = F(P/F, i\%, n)$$

The interest table approach to problem solving is usually less tedious and less time-consuming than using formulas, though not always as accurate.

**Be careful! and note the following to avoid making common mistakes**

- $P$ does not always mean present, *and* $F$ does not always mean future. Generally, $P$ and $F$ are relative to each other. $P$ is to the left of $F$ on a cash flow diagram, and $F$ is to the right. By extension, $P/F$ is used to move a value to a reference point in the past, and $F/P$ is used to move a value to a reference point in the future (see examples).
- Do not confuse when to use $P/F$ and $F/P$. Use fractions to remember when to use which one. When using $P$ to find $F$, set up the fraction that will cancel $P$ and leave $F$ and vice versa (see examples).
- Always use the formulas when asked for exact answers.
- The formula derived is for compounded interest. For simple interest, the equation reduces to:

$$F = P(1 + in) = P + Pin$$

## Compounding Interest Calculations

Traditionally, when we say the interest is 6%, we usually mean 6% per year even though it is not explicitly said. However, if the given interest rate has a specific compounding period other than annually, new measures must be taken in order to properly use the interest rate in the interest tables. The compounding period for interest could be semi-annually, quarterly, monthly, weekly, daily, and so on.

For example, when we say $i = 6\%$ compounded monthly, we must understand it as $i = 6\%$ per year, and we need to calculate the interest earned monthly (12 times a year). In this case, the value of $i\%$ to be used in our calculation will be found by dividing 6% by 12 to get 0.5% per compounding period (in this case per month). The $n$ value for our formula is then found by multiplying the number of years by 12 to get the number of months.

There is a big difference between 1% per month and 1% compounded monthly. If a problem clearly states that $i = 1\%$ per month, then this value can be used directly in the calculations.

This is extremely important to remember as it is where most people make a very serious financial mistake. See more examples on this topic in Table 2.1.

Given that $P = \$500$, $n = 3$ years, and $i = 6\%$ per year (but with different compounding periods), find $F$ when the compounding periods are as follows. Students complete the following calculation column.

**Table 2.1** Compound Interest Calculation

| | *i*% Used in Calculation | *n* Value Used in Calculation | *F* Used in Calculation | Calculation $F = P(1 + i)^n$ |
|---|---|---|---|---|
| **a) Annually or Yearly** | 6% | 3 × 1 = 3 | 595.51 | $F = 500\ (1 + 0.06)^3$ |
| **b) Semi-Annually** | 6%/2 = 3% | 3 × 2 = 6 | 597.03 | $F = 500\ (1 + 0.03)^6$ |
| **c) Quarterly** | 6%/4 = 1.5% | 3 × 4 = 12 | 597.81 | |
| **d) Monthly** | 6%/12 = 0.5% | 3 × 12 = 36 | 598.34 | |
| ***e) Weekly** | 6%/52 | 3 × 52 = 156 | 598.55 | |
| ***f) Daily** | 6%/365 | 3 × 365 = 1095 | 598.60 | |

*Please note that *i*% value is very sensitive to the final answer, therefore it is recommended to use the entire value of *i*% such as in the case of *e) and *f) above.

*For the value of i% we need at least* ***three decimal places*** *for the answers.*

For example: If the calculated $i = 0.1296551$

Then the correct answers can be:

$i = 12.96551\%$
or $i = 12.9655\%$
or $i = 12.966\%$

But the following answers are not acceptable:

$i = 12.97\%$
or $i = 13\%$

This will result in an error when dealing with a large amount of money.

## Nominal and Effective Interest

### Basic Concept

Interest rates that are for a time period and for a compounding subperiod other than one year require special attention.

### Details

The following is a list of necessary vocabulary.

- APR = annual percentage rate (announced *i*%/year)
- APY = annual percentage yield (actual earned *i*%/year)
- $r$ = nominal rate. This is the rate of interest *per year* which does not consider the effect of any compounding.
- $m$ = number of compounding subperiods *per year*
- $i$ = effective interest rate *per compounding period*
- $i_a$ or $i^*$ = effective annual interest rate

Note: "*Effective*" means "*Actual*"

$$i_a = (1 + \frac{r}{m})^m - 1 = (1 + i)^m - 1$$

Relationship between APR and APY:

- APR = APY when interest is compounded once a year.
- If interest is compounded more than once a year APY>>APR.

## Important Note

- APY and/or APR are often compared from one project to another in order to make a decision. The project with the highest APY would clearly be the most attractive, as would the loan or purchase with the lowest APR.

**Be careful! and note the following to avoid making common mistakes**

- Be sure to understand what the terms in this section actually represent.
  - *r* is the annual rate without considering the effect of compounding period.
  - $i_a$ or $i^*$ on the other hand is literally the effective rate when the effect of compounding is taken into account.
- To find *r*, multiply *i* (the actual interest rate/compounding period) by *m* to get the value of nominal rate of interest per year.
- Please pay attention and understand the real meaning of all symbols used ($i$, $i^*$, $r$, $m$) on this topic. This is very important in accounting and finance. Mistakes should not be made, or it may result in serious financial consequences.



### Example 2.1: Find the Interest Rate per Year Using the Formula $F = P(1+i)^n$

$F$ = 12,100; $P$ = 1000; $n$ = 22 years

$$F = P\,(1+i)^n$$

$$12{,}100 = 1000\,(1+i)^{22}$$

$$(1+i)^{22} = 12.1$$

$$\sqrt[22]{(1+i)^{22}} = \sqrt[22]{12.1}$$

$i = 0.1199999 \approx 0.12$

$i = 12\%$

### Example 2.2: Using the Interest Table to Find the Interest Rate

$F$ = 1710; $P$ = 1000; $n$ = 11 years

$$F = P(F/P, i\%, n)$$

$$1710 = 1000\,(F/P, i\%, 11)$$

$$(F/P, i\%, 11) = 1.710$$

From the interest table, find $i = 5\%$

**Example 2.3: Interest Rate per Month, Nominal Rate, and Effective Rate**

A credit card company compounds the interest monthly after 3 years of your debt. You have to pay off your debt of \$2040 ($F$), from $P$ = \$1000 only. Find:

**a.** $i$%/month **b.** $r$ **c.** $i$ (effective rate per year)

**a.** $F = P\,(1+i)^n$

$$2040 = 1000\,(1+i)^{3 \times 12}$$

$$2.040 = (1+i)^{36}$$

$$(1+i) = \sqrt[36]{2.040} = 1.02000$$

$$i = 0.02000 \approx 2\%$$

Thus, $i$ = 2% per month = *effective or actual rate/month*

**b.** So, $r = 2\% \times 12 = 24\%$ per year

$$i^* = \left(1+\frac{r}{m}\right)^m - 1$$

$$i^* = (1+i)^m - 1$$

$$i^* = (1+.02)^{12} - 1 = (1.02)^{12} - 1$$

$i^* = 0.26824 = 26.824\%$ per year

Remember to always use three decimal places. The following answers are wrong: 26.8%, 26.82%, 27%.



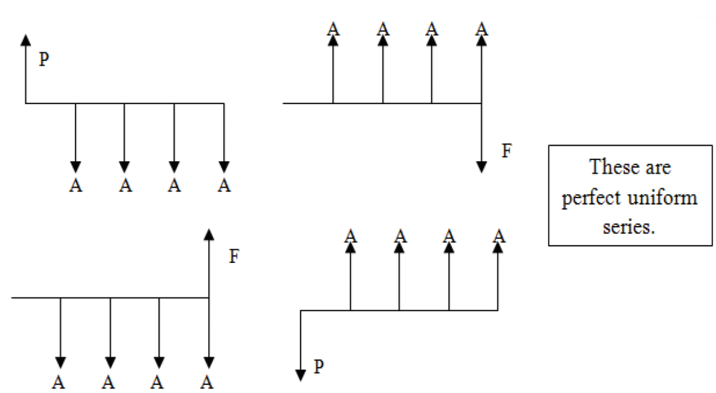
## Uniform Series Compound Interest Formula

### Basic Concept

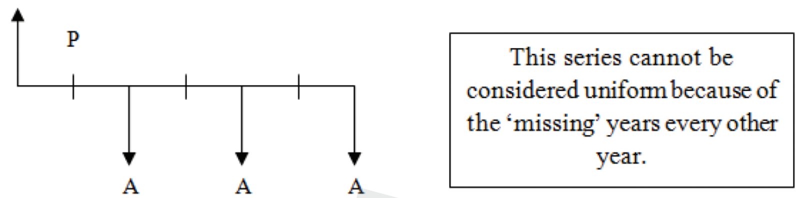
When a series of uniform end-of-period cash disbursements appear in a cash flow diagram, a special formula can be used to consolidate them into a $P$ or $F$ value.

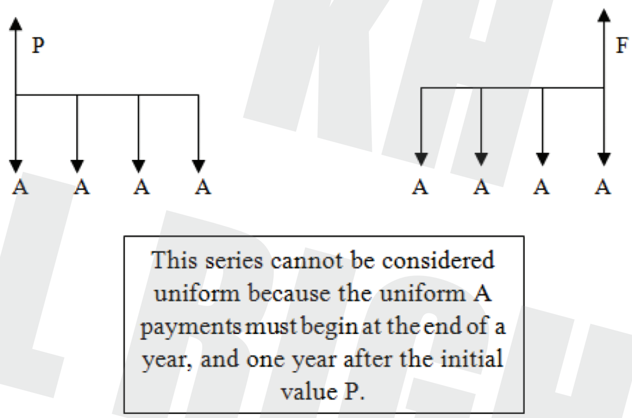
### Details

The following is a set of examples designed to show what constitutes a uniform series.



The following examples show what does not constitute a uniform series. Therefore, these cash flows cannot directly use the formula for a uniform series.






- To find $F$ given $A$, refer to the interest table for the given rate and locate the proper number of years under the $F/A$ heading. Multiply $A$ by this number (the part of the formula in parentheses) to find $F$ as is shown in the following formula:

$$F = A(F/A, i\%, n)$$

- To find $P$ given $A$, refer to the interest table for the given rate and locate the proper number of years under the $P/A$ heading. Multiply $A$ by this number (the part of the formula in parentheses) to find $P$ as is shown in the following formula:

$$P = A(P/A, i\%, n)$$

This approach can be used to consolidate a series of uniform payments into a single point. That single value can then be used as a $P$ or $F$ in a further calculation.

$A$ can be found given $F$ or $P$, using the $A/F$ and $A/P$ heading, respectively. This can be seen in the following formula to find $A$ given $F$:

$$A = F(A/F, i\%, n)$$

Similarly, to find $A$ given $P$ use the following formula:

$$A = P(A/P, i\%, n)$$

**Be careful! and note the following to avoid making common mistakes**

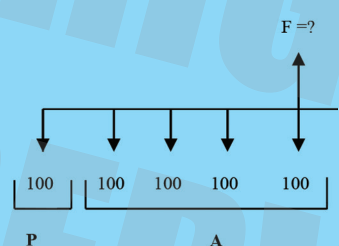
- Always check that the cash flow diagram you are using matches one of the major original diagrams (from which the formulas are derived) if you need to use the uniform series concept. Specifically, check the locations of *P* or *F*. Because A is always at the end of the period, *P* always appears to be one year before *A* begins, while *F* is on the same year as the last *A*.
- If at some point you need to use one of these formulas, and the first transaction in a uniform series is located at the beginning of the first year, simply imagine a year before it, and add that space to your cash flow diagram as a dotted line. The first *A* must occur at the end of a period (see Example 2.5).
- Again, note that the "canceling fractions" tip for recognizing whether to use one combination of variables or another still holds here (*A*/*P* or *P*/*A*, etc.).



### Example 2.4: Find *F* Given Uniform Payments Using Different Methods

Consider the cash flow diagrams given. Use $i$ = 10% and find *F*.

***Method 1***




$F1 = A$ and $F2 = P$

$F = F1 + F2$

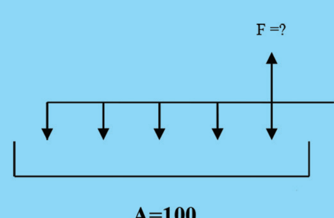
$= A\ (F/A, 10\%, 4) + P\ (F/P, 10\%, 4)$

$= 100\ (4.641) + 100\ (1.4641)$

$= 464.1 + 146.41$

$= 610.51$

***Method 2***




Note that we use 5 periods since *A* is at the end of the period.

*Continued*

$F = A\ (F/A,\ 10\%,\ 5)$
$= 100(6.105)$
$= 610.5$

Results in the same answer as Method 1!
Four important points should be noted in the derivation and the use of interest factors:

1. The end of one year is the beginning of the next year.
2. *P* is at the beginning of a year at a time regarded as being the present.
3. *F* is at the end of the *n*th year from a time regarded as being the present.
4. "A" occurs at the end of each year of the period under consideration.

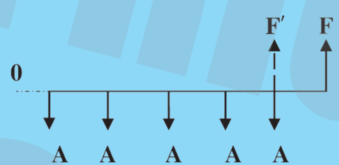
When *P* & *A* are involved, the first *A* of the series occurs one year after *P*. When *F* & *A* are involved, the last *A* of the series occurs at the same time as *F*.



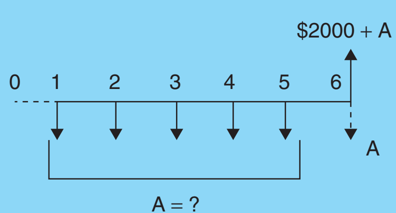
## Example 2.5: Finding *A* Given *F* Using Different Methods

What amount must be deposited at the beginning of each year for the next 5 years into a savings account that earns 10% interest, in order to accumulate $2000 at the end of 5 years?

***Method 1***




$F' = A\ (F/A,\ 10\%,\ 5)$
$F = F'\ (F/P,\ 10\%,\ 1)$
$F = A\ (F/A,\ 10\%,\ 5)\ (F/P,\ 10\%,\ 1)$
$A = F/[(F/A,\ 10\%,\ 5)\ (F/P,\ 10\%,\ 1)]$
$A = F\ (A/F,\ 10\%,\ 5)\ (P/F,\ 10\%,\ 1)$
$A = 2000\ (0.1638)\ (0.9091)$
$A = 2000\ (0.1489)$
$A = \$297.813$




***Method 2***

$A = F\ (A/F,\ 10\%,\ 6)$
$A = (2000 + A)\ (0.12961)$
$A = \$297.820$

*Continued*

Summary of A's characteristics

1. A is a constant value
2. A is an end-of-period transaction
3. A continues for $n$ periods

When N = ∞ (A continues forever):

$$A = P\left[\frac{i\,(1+i)^n}{(1+i)^n - 1}\right] = P(A/P\ i\%\ n)$$

When $n \to \infty$, then $(1+i)^\infty \approx ((1+i)^\infty - 1)$

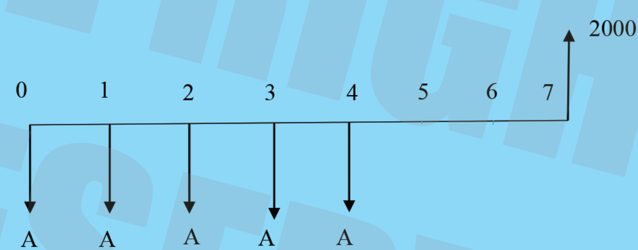
$$A = P\left[\frac{i\,\cancel{(1+i)^\infty}}{\cancel{(1+i)^\infty - 1}}\right] = P \times i$$

$A = P \times i$, when $n \to \infty$



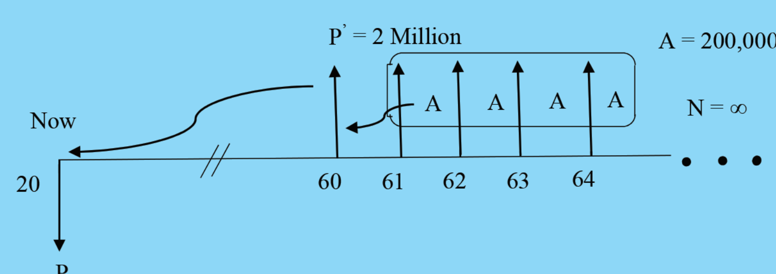
## Example 2.6: Drawing Cash Flow Diagrams

You deposit an equal amount of "A" at the beginning of each year for the next 5 years, and then you stop. At the end of year 7, you withdraw all of the money and get $2000. Draw a cash flow diagram that represents this scenario.




## Example 2.7: Retirement Plan

You are 20 years old now and plan to retire when you turn 60. Starting at the end of 61 years of age, you need $200,000 per year for the rest of your life. Assuming that you can manage to gain 10% of interest for a very long period of time from the investment, find the initial investment amount you need to invest now (at the present time). Draw a cash flow diagram and solve for $P$.




$A = P' \times i$

$P' = A/i = 200{,}000/0.1 = 2$ Million

$P = F\ (P/F,\ 10\%,\ 40) = 2{,}000{,}000\ (P/F,\ 10\%,\ 40) = \$44{,}200$

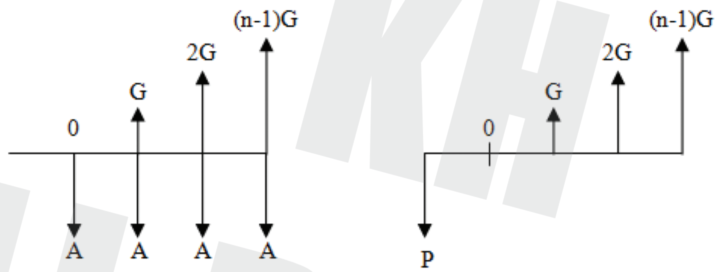
## Arithmetic Gradient Uniform Series

### Basic Concept

When a series of payments in a cash flow diagram increases uniformly by a fixed amount, a new formula can be used to find $P$ or $F$ for that set. This new formula is called the arithmetic gradient.

### Details

The following are examples of a proper arithmetic gradient:




The arithmetic gradient formulation follows the previously established pattern. To find the value of the payments in the uniform series, $A$, which corresponds to a given arithmetic gradient, $G$:

- To find $A$ given $G$, locate the appropriate value from the interest table for the rate specified in the problem under the $A/G$ heading. Multiply the $G$ value by this number to find $A$ as is shown in the following formula:

$$A = G(A/G, i\%, n)$$

To find the initial value P, that results from the arithmetic gradient G:

- To find $P$ given $G$, multiply the appropriate value from the interest table for the rate specified in the problem under the $P/G$ heading. Multiply this number by $G$ to find $P$ as is shown in the following formula:

$$P = G(P/G, i\%, n)$$

The arithmetic gradient approach is important because it rounds out the basic skill set necessary for solving more complex problems in the later lessons.
**Be careful! and note the following to avoid making common mistakes**

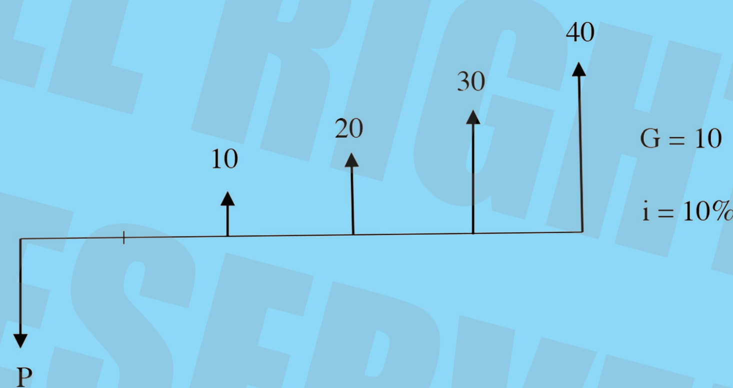
- The standard pattern of transaction (see Appendix A) for the arithmetic gradient is trickier than previous relations. *It is crucial to remember that the gradient begins after an empty period.* Remember this by picturing that the gradient must begin with a zero as the initial value and then it increases in equal increments. It is also important to remember that this empty period comes at the end of the year where $P$ is located.
- The arithmetic gradient must be *increasing* with time.

- A series of payments that closely resembles an arithmetic gradient can often be made to fit the skeleton by employing the "net" principle discussed earlier. For example, the following cash flow diagrams are equivalent. The first has been rewritten as the second in order to make it solvable using the arithmetic gradient approach. The positive arrows have a value *A* and the negative arrows have values *G*, 2*G*, and 3*G*. Note that nothing has actually been added or removed because each year retains the same net value, and that while the gradient must be increasing, it need not necessarily be positive.




### Example 2.8: Gradient Cash Flow

Consider the cash flow diagram given below. If $i$ = 10%, find $P$.

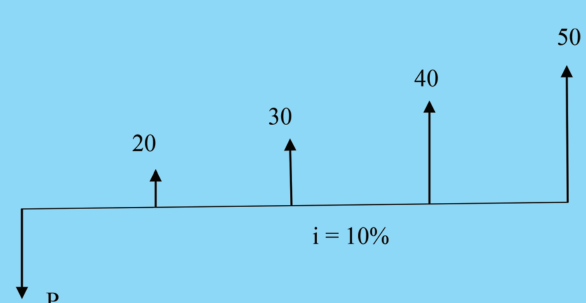



$P = G\left({}^{P}\!/_{G}, i\%, n\right)$

$P = 10\left({}^{P}\!/_{G}, 10\%, 5\right) = 10\,(6.862) = 68.62$

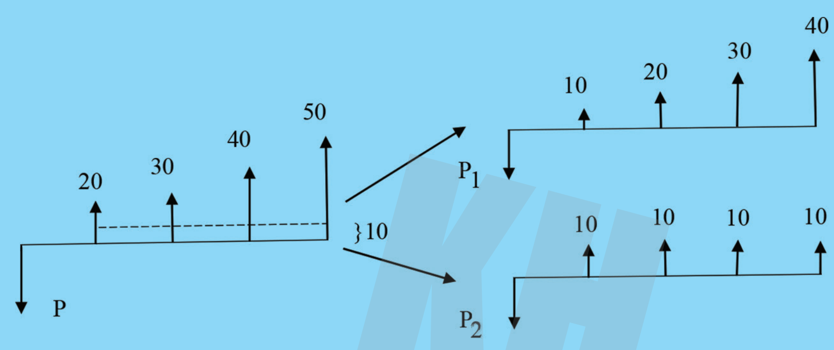
### Example 2.9: Gradient Cash Flow

Consider the cash flow diagram given below. If $i$ = 10%, find $P$.




*Continued*

Since the first transaction does not start with $G$ (value of 10), then we cannot get the $P$ value directly from the formula/direct relationship as shown in Example 8. In this case, the given cash flow diagram must be modified in such the way that the applied direct relationship is possible as follows:




$$P = P_1 + P_2$$

$$P_1 = 10\,(P/G, 10\%, 5)\,(F/P, 10\%, 1) = 75.48$$

$$P_2 = 10\,(P/A, 10\%, 4) = 31.70$$

Therefore, $P = 75.48 + 31.70 = 107.18$

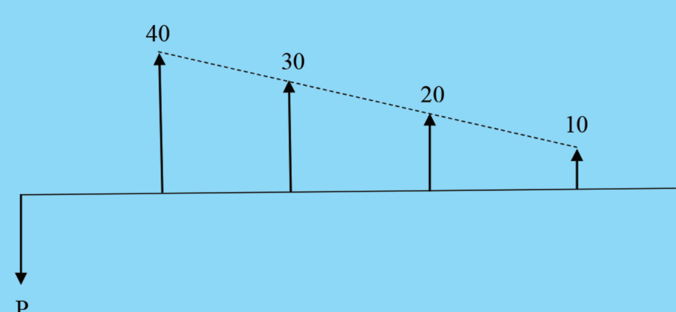
## Important Notes

What we learned from the earlier example is that we can modify the given cash flow diagram as long as no values are changed. The goal is to make the original cash flow diagram perfectly match the standard patterns, so that the direct relationship/formulas for those factors ($P$, $F$, $A$, $G$) may be easily applied. The direct relationship among $P$, $F$, and $G$ can be applied if and only if:

1. The first transaction of the series occurs at the end of the second period.
2. The first transaction must be equal to the $G$ value.

### Example 2.10: Decreasing Gradient

Consider the cash flow diagram given below. If $i = 10\%$, find $P$.

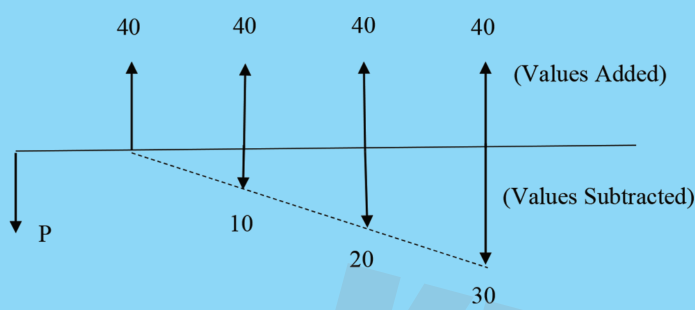



Continued

Lesson Two: Interest Formulas and Their Applications from Understanding Engineering Economy: A Practical Approach
by Khokiat Kengskool | 9781524917906 | 2016 Copyright | Property of Kendall Hunt Publishing



Step 1: Cash flow diagram modification without changing the original values.




Step 2: Solve for *P* based upon the standard relationships.

$$P = 40\,(P/A, 10\%, 4) - 10\,(P/G, 10\%, 4)$$
$$= 40\,(3.170) - 10(4.378)$$
$$= 83.02$$



## A Short-Cut Procedure to Find an Unknown Value

The following is an excellent short-cut procedure to find an unknown value from any equivalent cash flow diagram:

1. Understand the problem statement.
2. Convert step 1 into an accurate cash flow diagram.
3. Understand the structure of each standard cash flow diagram and its associated formula.
4. In **any equivalent cash flow diagram**, an unknown value can be easily found using the following approach:
   a. Select a convenient reference point within the cash flow diagram. Pick a point that will allow you to easily get the desired value from the most transactions.
   b. Transform or find the equivalent value of all transactions (*P*, *A*, *F*, *G*) from any point in time at the reference point.
   c. Remember that any positive transactions (arrow pointing up) or negative transactions (arrow pointing down) are still pointing in the same direction at the reference point. This is because you are simply finding the equivalent value at the selected reference point.
   d. Use the equation stating that the sum of all positive transactions at the reference point is equal to the sum of all negative transactions at the reference point. This can be written as follows:

$$\Sigma\uparrow = \Sigma\downarrow$$

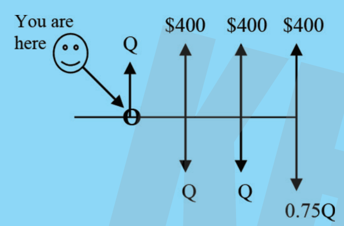
   e. Solve for the unknown value in the earlier equation.

### Example 2.11: Proof of Consistency when Using Different Reference Points

Consider the given cash flow diagram. If $i$ = 12%, find the value of Q using different reference points using $\sum\uparrow = \sum\downarrow$.

***Solution 1:***

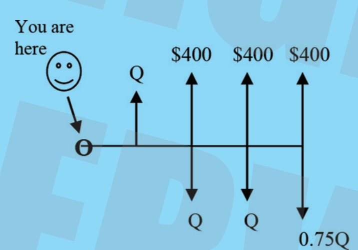
Use the Present as Reference Point




$$Q + 400\ (P/A,\ 12\%,\ 3) = Q\ (P/A,\ 12\%,\ 2) + 0.75Q\ (P/F,\ 12\%,\ 3)$$
$$Q + 400(2.402) = 1.690Q + 0.75Q\ (0.7118)$$
$$Q + 960.8 = 2.224Q$$
$$960.8 = 1.224Q$$
$$Q = \$784.97$$

***Solution 2:***

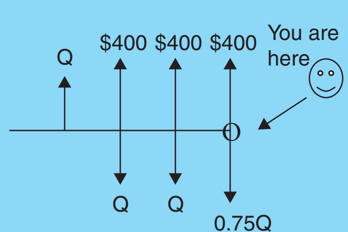
Use the First Year as Reference Point




$$Q\ (P/F,\ 12\%,\ 1) + 400\ (P/A,\ 12\%,\ 3)\ (P/F,\ 12\%,\ 1) = Q\ (P/A,\ 12\%,\ 2)\ (P/F,\ 12\%,\ 1) + 0.75Q\ (P/F,\ 12\%,\ 4)$$
$$0.8929Q + 400(2.402)\ (0.8929) = 1.509Q + 0.4766Q$$
$$857.90 = 1.093Q$$
$$Q = \$784.90$$

***Solution 3:***

Use the Last Year as Reference Point




*Continued*

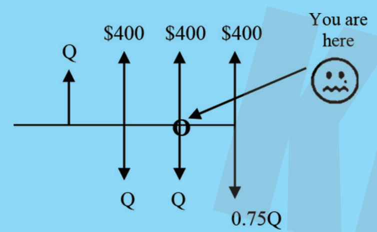
Lesson Two: Interest Formulas and Their Applications from Understanding Engineering Economy: A Practical Approach
by Khokiat Kengskool | 9781524917906 | 2016 Copyright | Property of Kendall Hunt Publishing



$$Q\ (F/P,\ 12\%,\ 3) + 400(F/A,\ 12\%,\ 3) = Q\ (F/A,\ 12\%,\ 2)\ (F/P,\ 12\%,\ 1) + 0.75Q$$
$$1.405Q + 1349.6 = 2.3744Q + 0.75Q$$
$$1.405Q + 1349.6 = 3.1244Q$$
$$1349.6 = 1.719Q$$
$$Q = \$785.11$$

***Solution 4:***

Use the Second Year as Reference Point



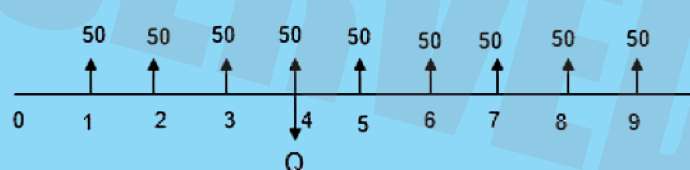

This is not a good reference point, but we still can get the right answer.

$$Q\ (F/P,\ 12\%,\ 2) + 400(F/A,\ 12\%,\ 2) + 400(P/F,\ 12\%,\ 1) = Q\ (F/A,\ 12\%,\ 2) + 0.75Q\ (P/F,\ 12\%,\ 1)$$
$$1.254Q + 848 + 357.2 = 2.12Q + 0.6697Q$$
$$1205.2 = 1.536Q$$
$$Q = \$784.80$$

Note: This example is to prove that any selected reference point will give the same similar answer.

## Example 2.12: Solve for Unknown Value Q

At the end of year 1 through 9, annual income = \$50. At the end of year 4, there is a major expense of \$Q. Draw this cash flow diagram and find Q if $i$ = 18%.




Use reference point at #4 and apply $\rightarrow \sum\downarrow = \sum\uparrow$

$$Q = 50\left(\frac{F}{A},18\%,4\right) + 50\left(\frac{P}{A},18\%,5\right) = \mathbf{417.10}$$

Alternatively, use reference point at 0.

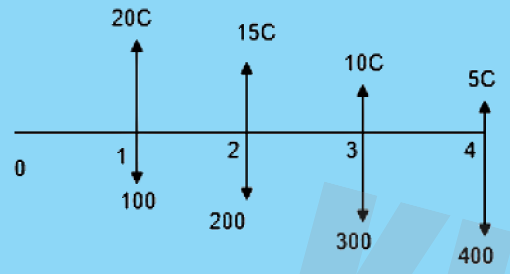
$$50\left(\frac{P}{A},18\%,9\right) = Q(\frac{P}{F},18\%,4)$$

$$Q = \mathbf{417.12}$$

Note that you get the same answer either way.

### Example 2.13: Solve for Unknown Value C

From the following equivalent cash flow diagram, find $C$ if $i = 12\%$.



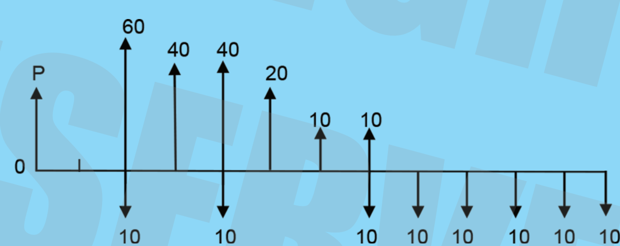

Use reference point at time 0 and apply $\rightarrow \sum\uparrow = \sum\downarrow$

$$20C\left(\frac{P}{A}, 12\%, 4\right) - 5C\left(\frac{P}{G}, 12\%, 4\right) = 100\left(\frac{P}{A}, 12\%, 4\right) + 100\left(\frac{P}{G}, 12\%, 4\right)$$

$$20C(3.037) - 5C(4.127) = 100(3.037) + 100(4.127) \rightarrow C = 17.87$$

### Example 2.14: Solve for Unknown Value *P*

If $i = 10\%$, find $P$ value from the given equivalent cash flow diagram presented below:




Use reference point at time 0 and apply $\rightarrow \sum\uparrow = \sum\downarrow$

$$P + 50\,(P/A, 10\%, 6)\,(P/F, 10\%, 1) = 10\,(P/G, 10\%, 6)\,(P/F, 10\%, 1) + 10(P/A, 10\%, 5)\,(P/F, 10\%, 7)$$
$$P + 50\,(4.355)\,(0.9091) = 10\,(9.684)\,(0.9091) + 10\,(3.791)\,(0.5132)$$
$$P + 197.96 = 88.04 + 19.46$$
$$P = -90.46$$

Note that the negative answer implies that the given $P$ value must actually be located in the opposite direction of the original at the same location. This is to keep the entire cash flow diagram perfectly balanced (equivalent cash flow diagram). An example of this is you perfectly paying off your loan for a certain number of years along with the interest involved.

## Summary

- Money has a time value because it is valuable and powerful.
- $P$ does not always mean present, and $F$ does not always mean future. These symbols are relative to each other on the timeline with $P$ always occurring first.

- Use interest tables to perform calculations quickly.
- To find $F$ given $P$, use the formula:

$$F = P(F/P, i\%, n)$$

- To find $P$ given $F$, use the formula:

$$P = F(P/F, i\%, n)$$

- For simple interest calculations, use the formula:

$$F = P(1 + i\ n)$$

- If not otherwise specified, assume that the interest given is compounded per year.
- To find the effective annual interest rate, use the formula:

$$i_a = (1 + \frac{r}{m})^m - 1$$

- APR equals APY when interest in compounded annually.
- $A$ is a constant value that occurs at the end of the period.
- $F$ is on the same year as the last $A$. To find $F$ given $A$, use the formula:

$$F = A(F/A,\ i\%,\ n)$$

- $P$ is always one year before $A$ begins. To find $P$ given $A$, use the formula:

$$P = A(P/A, i\%, n)$$

- $G$ increases uniformly by a fixed amount with time. Note that the first transaction of the series must be equal to the $G$ value and occurring after two empty periods.
- To find one value ($P$, $F$, $A$, or $G$) given another value, think of the formulas in the form of cancelling out fractions. The given value is in the denominator and visually cancels out to leave the desired value.
- A short-cut procedure presented in this lesson is an excellent tool to find an unknown value in any equivalent cash flow diagram. In this method we apply the equation:

$$\Sigma\uparrow = \Sigma\downarrow$$

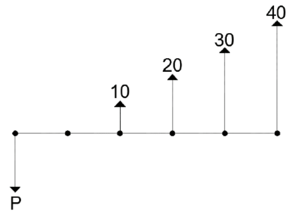
The sum of all positive transactions at the reference point equals the sum of all negative transactions at the same reference point. With one equation and one unknown, the needed answer can easily be found.

- Remember that the selected reference point can be any point in time. However, it is highly recommended that you pick a point that can help you solve for the unknown value in the most convenient way possible based upon the standard pattern of transaction used to derive the interest formulas. Please see "Formula Summary Table" in Appendix A for reference.

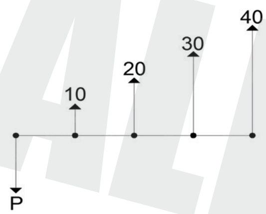
# Review Exercises

## Set 1: Problem Solving

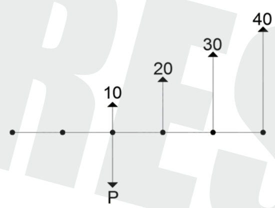
1. *Arithmetic Gradient:* Use $i = 10\%$ for all following problems.
   Find $P$ for parts a through d and find $Q$ for part e.



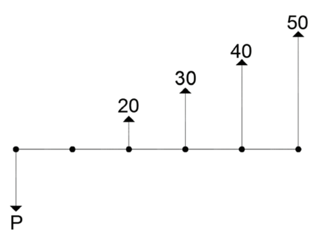

**a.**



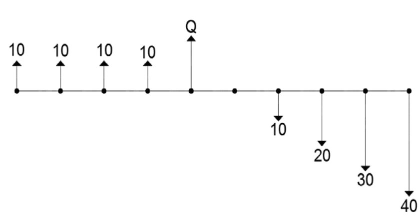

**b.**




**c.**




**d.**




**e.**

2. *Cash Flow Diagram*:
You deposit \$1500 per year at the beginning of each year for the next 3 years. By the end of year 5, F = \$6000. Draw the corresponding cash flow diagram that can represent these transaction activities.

3. *Effect of the Different Compounded Interest on the Future Value*:
You deposit \$3500 today at a local bank. If the bank gives you the interest of:
   a. 8% compounded monthly, find $F$ at the end of year 5.
   b. 8% compounded daily, find $F$ at the end of year 5.
   c. 8% annually with simple interest calculation, find $F$ at the end of year 5.

4. *Find i%*:
If you can get $F = \$5700$ from $P = \$2000$ for $n = 15$ years.
   a. Use the formula.
   b. Use the interest tables.

5. *Find n*:
If you can get F = \$11,500 from P = \$877.9 and i = 10% (meaning 10% per year), what should be the number of years for this transaction?
   a. Use the formula.
   b. Use the interest tables.

6. *Refinancing:*
You borrow \$5000 from a bank with an initial interest rate of 10% and promise to pay it back at the end of each year in equal amounts ($A$) for 10 years. After the first 3 years of payments, the bank changes the interest rate to 8% and then to 6% for the last four years.
   a. Find $A_1$ for the first 3 years.
   b. Find $A_2$ for years 4–6.
   c. Find $A_3$ for the last 4 years or for years 7–10.

7. *Budget Planning:*
The City of Miami wants to enhance a public park. According to the plans for this project, its initial cost is \$300,000 and the maintenance cost is \$12,000 per year. We assume that this park will last forever or $n$ = infinity. Use $i = 10\%$.
   a. Find the total budget that can support this project.
   b. Find the budget that will be used only for the maintenance cost.

8. *Retirement Plan:*
Today you are exactly 20 years old and this is your plan: At your 50th birthday (or exactly 30 years from now) you want to stop working completely. To maintain the same lifestyle, you must have an income of \$100,000 per year for the rest of your life starting from your 51st birthday. Your uncle is a banker and promises that he will manage your money to earn 8% interest per year for as long as you live.

a. How much money must you deposit into your savings account now (on your 20th birthday), so that your wish will come true?

Note: Remember after you retire, you want to be earning $100,000 per year, and we assume that this amount will last forever *(Use n = infinity as an assumption for the retirement plan).*

b. On your 120th birthday, you decide to withdraw all of your money from the savings account. How much would you receive at that time?

## Set 2: Multiple Choices Questions

1. A college student takes out a loan for $50,000 to pay for school. She wants to pay it back within 4 years by making equal monthly payments. There is a 12% interest rate on the money. How much will her monthly payments be?
   A. $1315
   B. $3492
   C. $6035
   D. $7426

2. You want an investment that will pay out $7000 at the end of each year for the next 5 years. What do you have to invest today to make that possible? Assume a 6% interest rate.
   A. $24,365
   B. $27,937
   C. $29,484
   D. $36,458

3. You invest $200 today, $300 next year, and $400 three years from now. How much money will you have 15 years from now if you are earning 10% interest?
   A. $3959
   B. $3229.70
   C. $6003
   D. $7773

4. Given an effective annual interest rate of 6%, what is the nominal interest rate compounded quarterly?
   A. 5.3%
   B. 5.9%
   C. 6.1%
   D. 6.5%

5. If you invest $5000 today, what is its future value after 10 years at an 8% interest rate?
   A. $5683
   B. $8769
   C. $10,795
   D. $13,594

6. A man is retiring with $250,000 in the bank. He plans on spending $30,000 a year for the rest of his life. What is the interest rate he needs to make this happen?
   A. 11.5%
   B. 12.0%
   C. 12.4%
   D. 12.9%

7. What is the present cost of a crane that has an initial cost of $60,000, an annual benefit of $5000, an annual maintenance cost of $1500, a *salvage value* of $8000 and a useful life of 7 years? Assume 12% interest rate.
   A. $35,575
   B. $37,972
   C. $38,274
   D. $40,408

   Note: *Salvage value* is the amount of money which is earned from selling the old asset at the end of its useful life or at the end of a project.

8. What is the rate of return (or % rate of interest) on $5000 today that is worth $10,000 in 15 years?
   A. 1.4%
   B. 2.6%
   C. 3.9%
   D. 4.7%

9. What is the future value of a $15,000 investment today if for the next 4 years the interest rate is assumed to be 5%, then 8% for the following 5 years, and then 10% for the last 6 years after that?
   A. $35,937
   B. $39,280
   C. $42,758
   D. $47,480



## Set 3: Multiple Choice Questions

1. A student obtained a loan of $1500 and agreed to repay it at the end of 5 years, together with 10% simple interest per year. How much will she pay 5 years hence?
   A. $2600
   B. $3120
   C. $2250
   D. $2100

**2.** A $6000 loan was to be repaid with 10% simple annual interest. A total of $6350 was paid. How long had the loan been outstanding?
   **A.** 1 year
   **B.** 5 months
   **C.** 8 months
   **D.** 7 months

**3.** A writer obtained a $1000 loan from the bank. He agreed to repay the sum at the end of 6 years, together with an interest of 8% per year. How much will she owe the bank at the end of 6 years?
   **A.** $1587
   **B.** $945
   **C.** $1200
   **D.** $1750

**4.** Suppose that $3500 is deposited into an account that earns 8% interest. How much is in the account after 7 years?
   **A.** $2676.45
   **B.** $6780.30
   **C.** $5998.40
   **D.** $3150.45

**5.** An amount of money will be $15,000 at an interest rate of 10%. How much is its present value if it will be received in 10 years?
   **A.** $8220
   **B.** $5783.15
   **C.** $6540
   **D.** $7200

**6.** Alice borrows $3000 from her friend. She repays him $4000. What was the interest rate if she paid the $4000 at the end of year 2?
   **A.** 10%
   **B.** 15.47%
   **C.** 11.56%
   **D.** 12.25%

**7.** How long would it take to double an amount of money *P*, if it is invested at 8% simple interest? And at 8% compound interest?
   **A.** 12.5, 9
   **B.** 8, 10
   **C.** 25, 17.6
   **D.** 20.3, 8

Lesson Two: Interest Formulas and Their Applications from Understanding Engineering Economy: A Practical Approach
by Khokiat Kengskool | 9781524917906 | 2016 Copyright | Property of Kendall Hunt Publishing



8. A savings account earns 7% interest. If $2000 is invested, how many years will it take for the amount on deposit to be $2360?
   A. 2.5
   B. 4.3
   C. 5.0
   D. 3.8

9. How much is required to be invested now at 10% interest to obtain $200,000 in 70 years?
   A. $1454
   B. $1200
   C. $845.67
   D. $253.25

10. The following series of payments will repay a present sum of $7000 at an 8% interest rate. Use single payment factors to find the present sum that is equivalent to this series of payments at a 12% interest rate.

| EOY | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cash Flow | $1400 | $1320 | $1240 | $1160 | $1080 |

   A. $4500.80
   B. $4535.24
   C. $4543.50
   D. $4434.87

11. What amount of money now is equivalent to $9500 two years later, if interest is 6% per 6-month period?
   A. $7525
   B. $7200
   C. $7120
   D. $7850

12. A sum of money invested at 6% per 6-month period (semiannually) will double in amount in approximately how many years?
   A. 5
   B. 7
   C. 6
   D. 10

Lesson Two: Interest Formulas and Their Applications from Understanding Engineering Economy: A Practical Approach
by Khokiat Kengskool | 9781524917906 | 2016 Copyright | Property of Kendall Hunt Publishing



## Lesson 2: Quiz Questions

**Name** ______________________ **ID#** ____________ **Date** ____________

**1.** Bank A offers to pay you 6% interest on savings deposits, while bank B will pay 1.5% per 3-month period (quarterly). You want to make a deposit of \$1340 to put into a savings account. Assuming you want to leave all money in the account for 3 years, how much additional interest would be obtained from bank B over bank A?

**A.** \$4 **B.** \$6 **C.** \$10 **D.** \$8 **E.** \$12

**2.** A sum of money, *Q*, will be received 3 years from now. At 10% annual interest, the present worth of *Q* is \$950. At the same interest rate, what would be the value of *Q* in 7 years?

**A.** \$977.30 **B.** \$804.30 **C.** \$1450.30 **D.** \$1851.30

**3.** You borrow \$10,000 to purchase a car. You have to repay the loan in 48 equal end-of-period monthly payments. Interest is calculated at 1.25% per month. Determine the following:

**3.1** The nominal interest rate is most nearly:
**A.** 15% **B.** 60% **C.** 1.5% **D.** 6% **E.** 20%

**3.2** The effective annual interest rate is most nearly:
**A.** 17.7% **B.** 15.05% **C.** 16.08% **D.** 15% **E.** 20%

**3.3** The effective rate of interest per month is most nearly:
**A.** 1.34% **B.** 1.25% **C.** 1.30% **D.** 1.52% **E.** 1.43%

**3.4** The amount of the monthly payment is most nearly:
**A.** 287 **B.** 300 **C.** 250 **D.** 270 **E.** 278

**4.** You borrow \$1000. To repay this amount you have to make 12 equal monthly payments of \$90.30. Determine the following:

**4.1** The effective monthly interest rate is most nearly:
**A.** 1.50% **B.** 1.20% **C.** 1.30% **D.** 1.25% **E.** 1%

**4.2** The nominal annual interest rate is most nearly:
**A.** 16% **B.** 15% **C.** 14% **D.** 13% **E.** 17%

**4.3** The effective annual interest rate is most nearly:
**A.** 16.80% **B.** 18.60% **C.** 18.06% **D.** 16.08% **E.** 18.86%