Chapter 3 - Deciphering the Code from Tobias/Piercey's Banishing Math Anxiety
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Chapter Three

Deciphering the Code

Mathematics has its own vocabulary. In addition to words, mathematics uses its own notation, symbols that stand for more complicated ideas. Some of these elements are familiar, such as numbers, letters, and the arithmetical operations signs like + and −. Some are less so, as in algebra, where mathematical notation also includes parentheses and brackets to show which operations are to be done in which order. Be on the lookout for new symbols in college math, including certain letters from the Greek alphabet. The notation may be "Greek to you," but it speaks volumes to people who understand it.

Mathematics is not the only discipline that uses its own notation. Music and dance do too. Just as a musician can read sheet music without having to translate the symbols into words, mathematicians read mathematical notation directly and without translation.

Our goal in this chapter is to get you to understand the code once you get out of the habit of translating mathematical statements into words.

First, let's do an experiment. Try "talking" this mathematical equation:

$$\$120/40 = \$3$$

In words, you could state that if 40 identically priced items together cost $120, then any one of those 40 items must cost $3. But when you have an expression like a/n where neither a nor n is expressible as a number or something concrete, you have to interpret the expression without using words. Learning to read notation without a word-for-word translation is important because a few pages later you will encounter a mathematical statement that wouldn't be at all easy to translate into words.

$$\frac{(a - V)v}{aTv - RT}$$

Just as an advanced student of a foreign language no longer has to translate word for word, with practice, you will be able to *think* in mathematical notation. That should be your goal.

Understanding Notation

Although the most familiar symbols in mathematics are numbers, letters are used as symbols as or more often than numbers. You met x and y in high school algebra as standing in for "unknowns." Very often one or the other was the answer to a problem you were supposed to solve. They will still be unknowns in college math, but with a meaning: *unspecified*, rather than unknown. They will be called *variables*. Letters at the beginning or the middle of the alphabet tend to represent the opposite of variables: unchanging numerical values. They are called *constants.*

Let's look at an example of how we use variables to express and describe mathematical relationships. Think about the cost of a ride in a cab. Typically, there is a "flag-drop" fee just for picking up a passenger and an additional cost per mile driven. In New York City, a cab driver will charge you a flag-drop fee of $2.50 plus $2.00 for each mile driven. In Chicago, you would pay a $2.25 flag-drop fee and $1.80 for each mile. In Los Angeles, the flag-drop fee is $2.85 and you'll pay $2.70 for each mile.

Let's use variables to describe the relationship between the cost of a cab ride in each city and the number of miles driven. The variable y stands for the total cost of a cab ride and x stands for the number of miles driven. In New York City, we multiply the per-mile rate of $2 by the number of miles, x, then add the flag-drop fee of $2.50:

$$y = 2x + 2.50$$

In Chicago, we multiply the per-mile rate of $1.80 by x and add the flag-drop fee of $2.25:

$$y = 1.80x + 2.25$$

Similarly, in L.A.:

$$y = 2.70x + 2.85$$

What do the equations for all three cities have in common? In each city, there is some unchanging number m (the per-mile rate) and another number b (the flag-drop fee). Using these *constants*, we can write:

$$y = mx + b$$

to express a general formula for the structure of cab fees.

We are typically only interested in the relationship between the cost of the cab ride and the number of miles driven. However, by not specifying the values for the constants m and b, we have a model that works for any cab ride in any city at any time!

One way you can recognize mathematical terms is to know which letters of the English alphabet have special mathematical meanings. Here are a few:

k: typically a constant

i: either an index or the imaginary number $\sqrt{-1}$

e: an irrational number (like π), $e \approx 2.718$

m: slope

f: the name of a function

The code extends to Greek letters, some of which have dedicated meanings. You have most likely seen π for the ratio of the circumference of a circle to the length of its diameter. Others include Σ (*s* in Greek) for summing a series of items and Δ (*d* in Greek) for change in a variable. Often clusters of these symbols are treated as single terms. Examples include Δh, $f(x)$, and $\log(x)$.

Just as in music and dance notation, mathematical notation packs a lot of content. For example, the concept of the logarithm involves four mathematical ideas.[1] All of these ideas are wrapped up neatly when we write $y = \log(x)$.

There is no need to memorize notation. Trust us. The more you see it and the more you use it, the more comfortable you will get. Even though mathematics is not a spoken language, it's helpful to practice speaking new terms out loud so you can "talk mathematics" when asking questions in class. Whenever a new term is introduced, whether in your text or in class, a full explanation will be given. How are you going to remember every new term? Now's the time to think about adding a glossary of notation to your glossary of words!

Subscripts and superscripts are an important part of mathematical notation. You have already learned that the superscript 2 in x^2 signifies x multiplied by itself. The subscript 1 as in x_1 is a way of creating more variables. Remember when there were two students named John in your elementary class? In order to remember who was who, the teacher usually appended the initial of their last names, "John P." and "John Q." This is exactly the same thing that is being done with subscripts. When two variables have the same *first* name (x), we append a subscript, like so: x_1 and x_2, read "x-sub-1" and "x-sub-2."

The key to mastering all of this is to take your time, and get help when you need it from your instructor, your tutor, or your study-mates. Get used to writing or keyboarding symbols. In short, take the new code seriously. We've used the foreign language analogy in this chapter. Mathematics is not technically a foreign language, but you would be wise to study its "vocabulary," its "idioms," and its "grammar" as though it were.

The Equals Sign and Its Relatives

One of the most important symbols in all of mathematics is the equal sign: =. You've met the equals sign before. But as you get into higher mathematics, it's important to pay closer attention to it. If an equals sign is found sandwiched between two expressions, it means that the expression on the left side of the equation has *exactly the same value* as the expression on the right side of the equation. For example, if we write:

$$2x + 4 = 8$$

we are communicating that the expression $2x + 4$ has the *same* value as the number 8 in this equation. This degree of equivalence also conveys that one side of the equation can be replaced with the other side.

[1] The notions of a function, exponential functions, one-to-one functions, and inverse functions are all wrapped up in the logarithm.

Some students confuse themselves by using the equals symbol to connect steps. This confusion leads to frustration. We have a solution if you feel that you need some way to connect your steps. How about using the symbol $\Rightarrow$?

$$2x + 4 = 8 \Rightarrow 2x = 4$$

Mathematics deals with approximate equality as well as exact equality. There's a difference in signs. The $=$ sign designates exactitude, and the $\approx$ sign designates approximate equivalents. The classic example of an approximate equivalent is the number π. You may remember from middle school math that the value of π is an unending decimal with no repeating pattern. When we use the decimal number 3.14 for π, we are acknowledging that 3.14 is an *approximation* because it would be impossible to write down an *exact* decimal equivalent. To convey this approximation, mathematicians write:[2]

$$\pi \approx 3.14$$

In addition to $=$ and $\approx$, you will encounter some other symbols that describe a relationship between two quantities. Here is a list:

$\neq$: What is on the left is *NOT* the same as what is on the right.

$>$: The value of the expression on the left is LARGER than that on the right.

$<$: The value of the expression on the left is SMALLER than that on the right.

$\geq$: The value on the left is either larger OR equal to that on the right.

$\leq$: The value on the left is either smaller OR equal to that on the right.

The Difference Between "Solve" and "Simplify"

What is the difference between the following assignments?

1. Simplify: $2x + 4 - (x + 1)$.
2. Solve: $2x + 4 = 8$.

The first involves an *expression.* The second involves an *equation.* An expression doesn't tell you very much. Think of it like a noun. If I write "a canine life form" by itself, I haven't really told you much. An equation, on the other hand, is a complete mathematical statement. The equals sign acts as the verb. As you will see, there is a lot more that you can do with an equation than with an expression.

When you are asked to *simplify* an expression, your teacher expects you replace the expression with an equivalent, but somehow simpler, notation. This would be like

[2] When students mistake decimal approximations for exact equality, they are frequently mixing up exactness with familiarity. Decimals are comfortable for us to use. They are concrete in the sense that decimal representations extend the place value system that we became comfortable with when learning about whole numbers. The value of 1.41 is clearer to most students than $\sqrt{2}$. There is nothing wrong with this and you have every right to feel this way. What we are suggesting is that you recognize that these values, although very close, are not exactly the same.

changing "the canine life form" to "the dog." In the math problem that we are given, we subtract the expression $x + 1$ from the expression $2x + 4$, resulting in $x + 3$.

Sometimes when comparing algebra problems, it may not be clear which expression is actually simpler. For example, which expression would you prefer to work with, $2x + 4$ or $2(x + 2)$? As you learn more math, you will see that which version of this expression you prefer to work with will depend on some larger problem that you are trying to solve. *Simplify* is one of those terms in math whose meaning is similar to, but not exactly the same as, the meaning in spoken English. What you are really doing when you are simplifying an expression is *rewriting that expression to make it more suitable to a given problem.*

But be careful. When you *simplify an expression,* you cannot change its value. You can only replace an expression with an equivalent expression.

When you are asked to *solve an equation* such as $2x + 4 = 8$, on the other hand, you are being asked to find certain unknown numbers. These numbers are values for the variable x. When you replace the variable in the expression $2x + 4$ with the correct values and compute, you will get the number 8. As we saw, this is what the use of the symbol $=$ demands!

Math instructors commonly use the analogy of scales and balances to differentiate solving and simplifying. Let's say you are asked to simplify an expression. If you think of the expression sitting on one side of a scale, when you replace it with an equivalent expression, the weight *cannot change*. When you are solving an equation, if you think about the two sides of the equation as being balanced on a see-saw, you are allowed to change the weight on each side, *provided the see-saw stays balanced*.

Piecewise Equations

In a higher-level math course, you may come across equations like the following:

$$y = \begin{cases} 2x, & 0 < x \leq 2 \\ 6 + 3(x - 2), & x > 2 \end{cases}$$

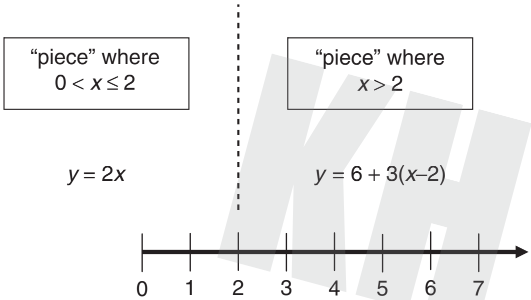
Translated into words the meaning is this: If you know x, you compute the value of y with one of two formulas. If x is less than or equal to 2, use the formula $y = 2x$, if x is larger than 2, use the other formula, $y = 6 + 3(x - 2)$. This is called a "piecewise" equation because it has two pieces.

Let's investigate this with a real-world application. Say you go to a parking garage. The garage charges \$3 per hour for the first 2 hours, and \$2 for each additional hour after that. How can we describe this situation in mathematical terms? We will use the variable x to stand for the number of hours we park, and the variable y to stand for the cost of parking.

There are two possibilities to consider: either we park for less than 2 hours or for more than 2 hours. If we park for less than 2 hours, then $y = 2x$. However, if we park for more than 2 hours, the equation is $y = 6 + 3(x - 2)$. What this means is we pay \$6.00 for the first 2 hours and \$3 for each additional hour.

The way to convey that our parking cost depends on whether x is smaller or larger than 2 is by using the earlier notation.

These types of equations are called "piecewise" because the first thing you have to do is determine which "piece" of the number line x falls on. Here is a visual way of thinking about this:



When the number of hours, x, that we park falls in the "piece" to the left of the vertical dotted line (where $x = 2$), we use the equation $y = 2x$ to compute the cost, whereas if x falls in the "piece" to the right, we use the equation $y = 6 + 3(x - 2)$.

Using the Code

Mathematical notation is a powerful code that allows its users to say a great deal in a very precise way without using much space. So in some ways, it's a language; in other ways it's a shorthand. Mastery, and eventually personal ownership, will unlock this code and enable you to apply mathematics to your class work and, as we shall see in Chapters 7 and 8, to your own life as well.